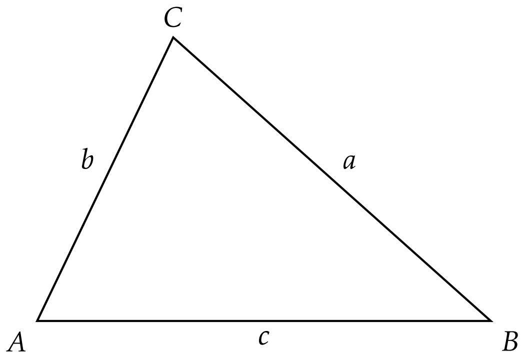
The Sine Rule

Labelling Convention



Capital letters for angles, lowercase for the side **opposite** that angle. So side a is opposite angle A.

Fact — **The Sine Rule:**

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use this when you have a **matching pair** — a side and the angle opposite it.

Example

In $\triangle ABC$, $a = 7$ cm, $A = 58°$, $B = 23°$. Find the length b.

Example

In $\triangle ABC$, $a = 7$ cm, $A = 128°$, $b = 6$ cm. Find angle B.

Example

In $\triangle ABC$, $a = 5$ cm, $A = 72°$, $C = 41°$. Find the length c.

The Cosine Rule

Example

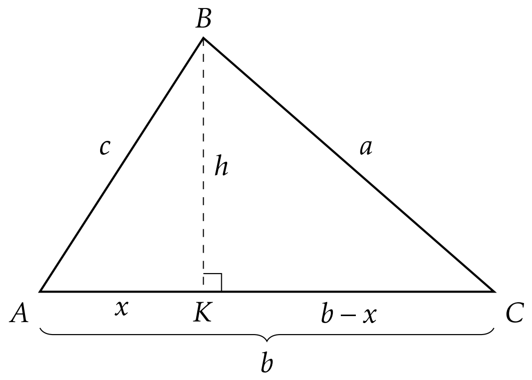
In $\triangle ABC$, $b = 8$ cm, $c = 5$ cm, $A = 60°$. Can you use the sine rule? Why or why not?

Fact — The Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Use this when you have **two sides and the included angle** (SAS), or **all three sides** (SSS).

Proof



Finding a side (SAS)

Example

In $\triangle ABC$, $b = 9$ cm, $a = 8$ cm, $C = 127°$. Find c.

Example

In $\triangle PQR$, $p = 11$ cm, $r = 7$ cm, $Q = 48°$. Find q.

Finding an angle (SSS)

Example

In $\triangle ABC$, $a = 8$, $b = 9$, $c = 10$. Find the largest angle.

Example

In $\triangle ABC$, $a = 5$, $b = 6$, $c = 9$. Find angle C.

Area of a Triangle: $\frac{1}{2}ab\sin C$

Example

You know two sides and the included angle of a triangle. Can you find its area without finding the height first?

Example

Find the area of $\triangle ABC$ where $a = 7$ cm, $b = 8.5$ cm, $C = 72°$.

Example

Find the area of $\triangle ABC$ where $a = 12.5$ cm, $A = 48°$, $B = 65°$.

Choosing the Right Rule

Fact — **Decision flowchart:**

1. Do you have a **matching pair** (side + opposite angle)?
   - Yes → **Sine Rule**
2. Do you have **two sides + included angle** (SAS)?
   - Yes → **Cosine Rule** (to find the third side)
3. Do you have **three sides** (SSS)?
   - Yes → **Cosine Rule** (to find an angle)
4. Always draw a diagram!
5. You may need to use $A + B + C = 180°$.

The Ambiguous Case of the Sine Rule

Example

In $\triangle ABC$, $a = 10$ cm, $b = 7$ cm, $A = 40°$. Find angle B.

Example

In $\triangle ABC$, $a = 5$ cm, $b = 8$ cm, $A = 30°$. Find angle B.

Fact — The ambiguous case occurs when using the sine rule to find an angle. Since $\sin\theta = \sin(180° - \theta)$, there may be **two valid solutions**. Always check whether the second solution gives a valid triangle (angles summing to less than 180°).